

Bit Error Rate Analysis of Block Coded Wireless System over α - μ Fading Channel

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Abstract: In this paper error performance of block coding such as systematic block coding and cyclic coding over α - μ fading channel is analyzed. In wireless channel error burst can be removed with the help of interleaver hence performance of random interleaver is also analyzed. Improvement in bit error rate (BER) performance after systematic block coding and cyclic coding with and without random interleaving, have been shown with simulation results.

Keywords: α - μ fading, systematic block coding, cyclic coding, interleaving, bit error rate.

I. INTRODUCTION

In wireless system error correction coding is applied to reduce the probability of bit or block error. The error correction enhances the performance on the cost of decrease in data rate or increase in signal bandwidth. In [6] design of coding schemes is considered for a channel affected by flat, slow fading and additive noise. Specifically, by using the block-fading channel model, the delay constraints have been taken into account. Optimum coding schemes for this channel model lead to the development of new criteria for code design, considering the minimum hamming distance. In [7] the problem of transmitting a Gaussian source on a slowly fading Gaussian channel, subject to the mean squared error distortion measure is discussed. Performance of reliable communication over non-coherent slow-fading multiple-input multiple-output channel at high signal-to-noise ratio is captured as a trade off between diversity and multiplexing gains. The problem of designing codes that optimally trade off the diversity and multiplexing gains have been studied in (8).

The paper is organized as follows. In Section 2, the alpha-mu fading model and its Probability density function is briefly discussed. In Section 3, block coding is discussed, systematic block coding and cyclic coding is explained. In Section 4, random interleaving technique is described. Monte-Carlo simulation results for BER performance for coded α - μ fading channel in different α and μ condition are presented in Section 5. The paper is concluded by Section 6.

II. THE α - μ FADING MODEL

The multipath fading in wireless communication is modelled by several distribution such as Rayleigh, Rician, Weibull, Nakagami. In the recent past alpha-mu (α - μ) fading model [1] has been proposed to describe the mobile radio signal considering two important phenomenon of radio propagation non-linearity and clustering. The α - μ represents a generalized fading distribution for small-scale variation of the fading signal in a non line-of-sight fading

condition. As given in its name, alpha-mu distribution is written in terms of two physical parameters, namely α and μ . The power parameter ($\alpha > 0$) is related to the non-linearity of the environment i.e. propagation medium, whereas the parameter ($\mu > 0$) is associated to the number of multipath clusters.

Table-1: Algorithm for generation of α - μ distributed random variable

1.	Procedure α - μ random variable generation
2.	$\alpha \leftarrow$ Channel Parameter
3.	$\mu \leftarrow$ Channel Parameter
4.	$n \leftarrow$ Number of random variables
5.	$m \leftarrow$ Mean
6.	$\sigma^2 \leftarrow$ Variance
7.	$H \leftarrow$ zero matrix of order $1 \times n$
8.	for $i \leftarrow 1$ to μ do
9.	$H = H +$ matrix of order $1 \times n$ having complex Gaussian random variable i.e. $N(m, \sigma^2) + j N(m, \sigma^2)$
10.	end for
11.	Fading envelope $\leftarrow H^\alpha$
12.	end procedure

In [1, 2] the α - μ fading distribution and its probability density function has been described. In the α - μ distribution, it is considered that a signal is composed of clusters of multipath waves. In any one of the cluster, the phases of the scattered waves are random and have similar delay times. Further, the delay-time spreads of different clusters is generally relatively large. As a result, the obtained envelope, is a non-linear function of the modulus of the sum of the multipath components.

The α - μ probability density function (PDF), $f_R(r)$ of envelope R is given as

$$f_R(r) = \frac{\alpha \mu^\mu r^{\alpha\mu - 1}}{\hat{r}^{\alpha\mu} \Gamma(\mu)} \exp\left[-\mu \frac{r^\alpha}{\hat{r}^\alpha}\right] \quad (1)$$

where $\alpha > 0$ is the power parameter, and α -root mean value of R^α is given as

$$\hat{r} = \alpha \sqrt{E(R^\alpha)} = \alpha \sqrt{2\mu\sigma^2}$$

where $\mu \geq 0$, is the inverse of the normalized variance of α - μ envelope R^α , and

$$\Gamma(z) = \int_0^\infty t^{z-1} e^{-t} dt \quad \text{is the Gamma function.}$$

Algorithm given in Table.1 is used for generation of α - μ distributed random variable in the simulation work reported in this paper. Analytical and simulated results for PDF of fading envelope of α - μ fading channel defined by eq. (1), are shown in Fig.1. It is verified that both the analytical and simulated results are matching.

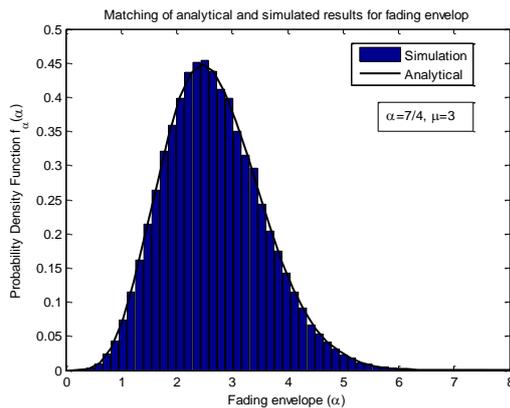


Fig.1. Matching of analytical and simulated results for α - μ fading envelope

III. BLOCK CODING

The increasing practical usage of digital communication systems has led of late to a great deal of work devoted to coding for fading channels. Coding in digital communication, allows bit errors introduced by transmission of a modulated signal through a wireless channel to be either detected or corrected by a decoder in the receiver. The general idea for achieving error detection and correction is to add some redundancy i.e. some extra data known as parity bits to a message, which receivers can use to check correctness of the transmitted data and to recover the same. The data bits along with the parity bits form a code word. The parity data are derived from the data bits by some deterministic algorithm. For error detection, a receiver can simply apply the same algorithm to the received data bits and compare its output with the received check bits, if the values do not match, an error has occurred at some point during the transmission.

A. Systematic Block Coding:

In systematic block codes one of the $M=2^k$ messages, each having binary sequence of length k , called message bits or information sequence, is mapped to a binary sequence of length n , called the codeword, where $n > k$. In an (n, k) binary block code, the rate of code is given as $R_c = k/n$ information bits per codeword symbol. If the codeword symbols are transmitted across the channel at a rate of R_s

symbols per second, then information rate associated with an (n, k) block code is $R_b = R_c R_s = (k/n)R_s$ bits per second. Thus the block coding reduces the data rate.

In a linear systematic block code, the codewords are generated from information bits with help of the generator matrix. A generator matrix is of the form given in eq.(2)

$$G = [I_k | P] = \begin{bmatrix} 1 & 0 & \cdots & 0 & p_{11} & p_{12} & \cdots & p_{1(n-k)} \\ 0 & 1 & \cdots & 0 & p_{21} & p_{22} & \cdots & p_{2(n-k)} \\ \vdots & \vdots \\ 0 & 0 & \cdots & 1 & p_{k1} & p_{k2} & \cdots & p_{k(n-k)} \end{bmatrix} \quad (2)$$

Where I_k is the $k \times k$ identity matrix and P is a $k \times (n-k)$ matrix that determines the redundant, or parity, bits to be used for error detection and correction.

The codeword output from encoder is of the form of eq.(3)

$$C_i = U_i G = U_i [I_k | P] = [u_{i1}, \dots, u_{ik}, p_1, \dots, p_{(n-k)}] \quad (3)$$

Where C represents the codeword, U the message and G the generator matrix. In eq. (3) the first k bits of the codeword are the original information bits and the last $(n-k)$ bits of the codeword are the parity bits.

B. Cyclic Coding:

Cyclic codes are a subclass of linear block codes in which all codewords in a given code are cyclic shifts of one another. The cyclic nature of cyclic codes creates a structure that allows their encoding and decoding functions to be simpler than the matrix multiplications associated with encoding and decoding for systematic block codes. The cyclic codes is extensively used in practice. Cyclic codes are generated via a generator polynomial instead of a generator matrix. The generator polynomial $g(X)$ for an (n, k) cyclic code has degree $n-k$ and is of the form of eq.(4)

$$g(X) = g_0 + g_1 X + \dots + g_{n-k} X^{n-k} \quad (4)$$

The codeword associated with a given k -bit information sequence is obtained from the polynomial coefficients of the generator polynomial multiplied by the message polynomial, thus the codeword is obtained from

$$c(X) = u(X)g(X) = c_0 + c_1 X + \dots + c_{n-1} X^{n-1} \quad (5)$$

As in systematic block codes, the first k codeword symbols equal to information bits and the remaining codeword symbols equal to parity bits, similarly a cyclic code can be put in the form of polynomial as shown in eq. (6), the first k bits as message bits from the polynomial, and the last $n-k$ bits in the codeword as parity bits.

$$p_0 + p_1 X + \dots + p_{n-k-1} X^{n-k-1} + u_0 X^{n-k} + u_1 X^{n-k-1} + \dots + u_{k-1} X^{n-1} \quad (6)$$

IV. RANDOM INTERLEAVING

Unlike wireline AWGN, in wireless due to multipath and fading, channel may exhibit bursty error characteristic, that is signal fading due to multipath propagation often causes the signal to fall below the noise level, resulting in large number of errors i.e. burst errors. Such error clusters are not usually corrected by codes designed for statistically independent errors. Suppose a burst of errors of length b has occurred which means a sequence of b -bit in errors. Then, the burst error correction capability of a systematic (n, k) code, which has $(n-k)$ parity check bits, is $b < \lfloor \frac{1}{2}(n - k) \rfloor$.

An effective method to improve performance of coding in fading channels for dealing with burst error channels is to interleave the coded data in such a way that the bursty channel is transformed to a channel having independent errors. Thus coding is typically combined with interleaving to mitigate the effect of error bursts.

The encoded data are processed by the interleaver and transmitted over the channel. At the receiver, the deinterleaver puts the data in proper sequence and passes them to the decoder. As a result of the interleaving/deinterleaving, error bursts are spread out in time so that errors within a code word appear to be independent. In a random interleaver, a block of N input bits are written in the interleaver in the same order in which they are received. Then, they are read out in a random manner. A random interleaver is a random permutation π . The interleaver has a corresponding deinterleaver (π^{-1}) that acts on the interleaved data sequence and restores it to the original order.

If the input data sequence is $U = [u_1, u_2, \dots, u_N]$, then permuted data sequence is $U \times P$, where P being the interleaving matrix with single 1, which is randomly located in each row and column, all other entries being zero. The de-interleaving matrix (P^T) is transpose of the interleaving matrix (P).

V. SIMULATIONS AND RESULTS

BER performance for systematic block coding, cyclic coding, and interleaving techniques over α - μ fading channel is obtained by Monte-Carlo simulation. The communication system shown in Fig.2 has been considered in this simulation. Results for different α and μ values are shown in Fig. 3 to Fig.10. In these simulation 500000 bits have been considered for a particular α and μ combination. The codeword $n = 7$, and each message is having binary sequence of length $k = 4$. The α - μ fading channel is assumed to be stationary for each codeword and changes independently for different codewords. In systematic block coding, generator matrix given in eq.(7) is used by encoder for generating codewords.

$$G_m = \begin{bmatrix} 1 & 0 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 1 & 1 & 1 \end{bmatrix} \quad (7)$$

In cyclic coding generator polynomial corresponding to matrix of eq. (8) is considered by encoder for generating codewords.

$$g_p = [1 \ 0 \ 1 \ 1] \quad (8)$$

In Fig.3 the BER performance for $\alpha=7/4$, $\mu=2$ is shown. It is noticed that BER performance for a given SNR improves after coding and further improves after interleaving. At 5 dB SNR, the uncoded BER is 0.2×10^{-2} , it improves after block/ cyclic coding to 4.5×10^{-3} , and further after interleaving in both the coding it is nearly 10^{-3} .

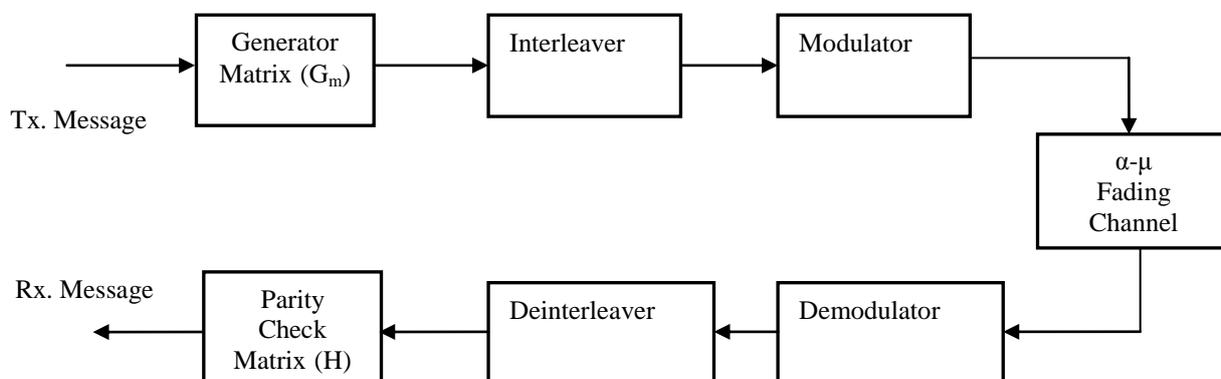


Fig. 2. Block coded communication system considered for simulation

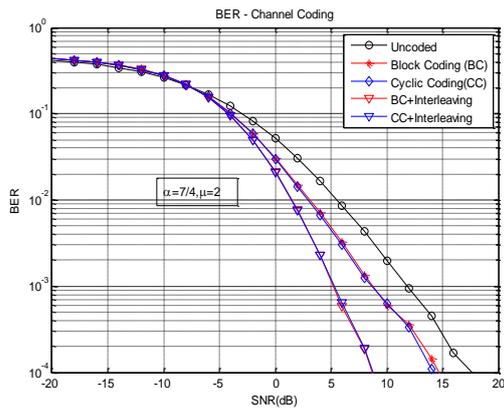


Fig. 3. BER after coding over α - μ fading for $\alpha=7/4$, $\mu=2$

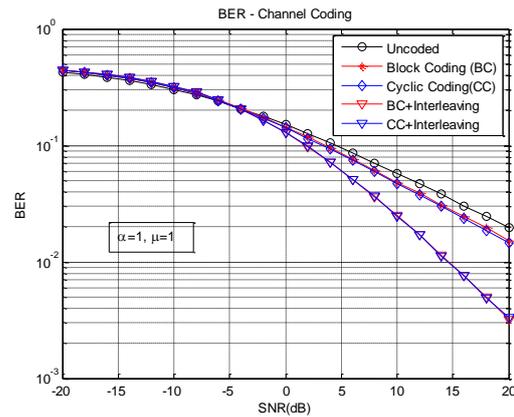


Fig. 4. BER after coding over α - μ fading for $\alpha=1$, $\mu=1$

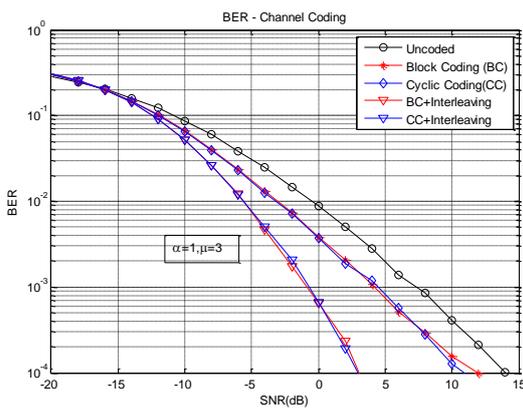


Fig. 5. BER after coding over α - μ fading for $\alpha=1$, $\mu=3$

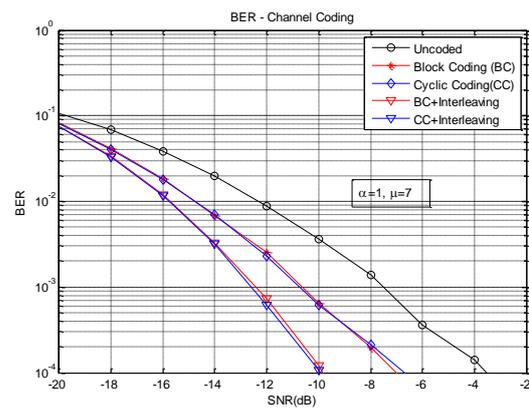


Fig. 6. BER after coding over α - μ fading for $\alpha=1$, $\mu=7$

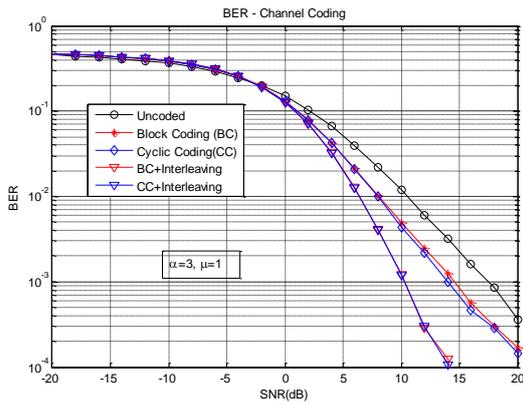


Fig. 7. BER after coding over α - μ fading for $\alpha=3$, $\mu=1$

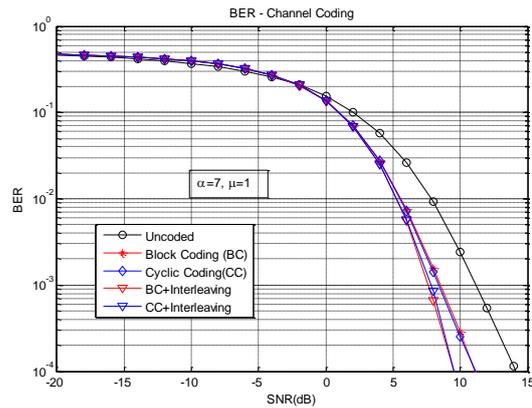


Fig. 8. BER after coding over α - μ fading for $\alpha=7$, $\mu=1$

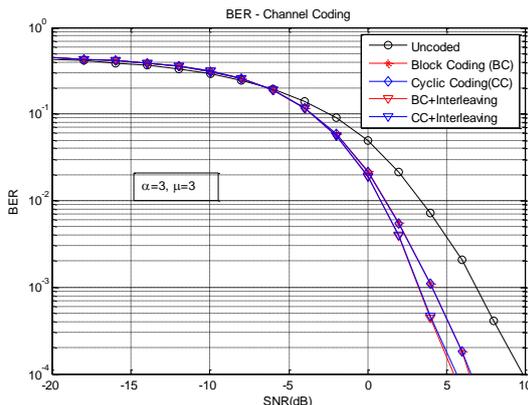


Fig. 9. BER after coding over α - μ fading for $\alpha=3$, $\mu=3$

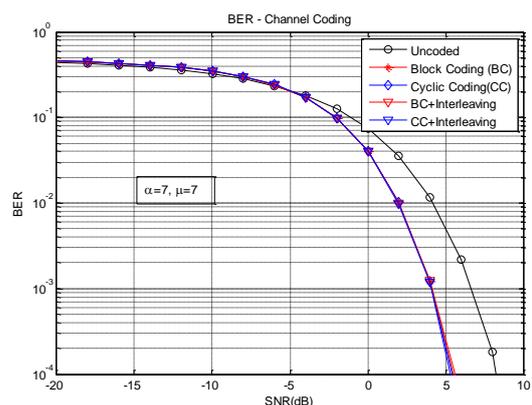


Fig. 10. BER after coding over α - μ fading for $\alpha=7$, $\mu=7$

In Fig.4 the BER performance for $\alpha=1$, $\mu=1$ is plotted. Here, it is observed, at 10 dB SNR, the uncoded BER is 6.0×10^{-2} , it improves after block/ cyclic coding to 5.0×10^{-2} , and further improvements of 2.5×10^{-2} is obtained after interleaving in both the coding. Fig.5 shows the BER performance for $\alpha=1$ and $\mu=3$, it is found that for 0 dB SNR, the uncoded BER is 9.0×10^{-3} , it improves after block/ cyclic coding to 4.0×10^{-3} , and further after interleaving in both the coding it is 6.0×10^{-4} . Similar, interpretation can be made from Fig.6, for $\alpha=1$, $\mu=7$, but since μ has increased, as anticipated improvement in BER can be observed than Fig. 5. Further, in Fig.7 the BER performance for $\alpha=3$, $\mu=1$ is given. It is observed, at 10 dB SNR, the uncoded BER is 1.2×10^{-2} , it improves after block/ cyclic coding to 4.5×10^{-3} , and further after interleaving in both the coding it is 1.4×10^{-3} . Similarly it is found that in Fig.8, for $\alpha=7$, $\mu=1$, now since α has increased, there is again improvement in BER observed than Fig. 7.

In Fig.9 the BER performance for $\alpha=3$ and $\mu=3$ is shown. It is found that for 5 dB SNR, the uncoded BER is 3.5×10^{-3} , it improves after block/ cyclic coding to 4.0×10^{-4} , and further after interleaving in both the coding it is 1.7×10^{-4} . Also, in Fig.10 the BER performance for $\alpha=7$, $\mu=7$ is given, where it is noticed, at 5 dB SNR, the uncoded BER is 5.0×10^{-3} , it improves after block/ cyclic coding and interleaving to nearly 2.0×10^{-4} . From the simulation results of Fig.4 and Fig.10, we find that in Fig. 10 where α and μ values are higher, the BER performance has significantly improved, due to high power and large number of multipath clusters.

VI. CONCLUSION

In this paper α - μ fading channel has been briefly introduced. BER performance of systematic block coding & cyclic coding, random interleaving are analysed. The simulated results of BER performance over α - μ fading for systematic block coding, cyclic coding with and without random interleaving schemes have been illustrated. The effect of α and μ parameters on BER performance is brought out. It is seen that the block coded wireless link outperform uncoded wireless link with a large margin. Link performance is further enhanced with the help of random interleaver. The improvement in performance due to coding is at the cost of reduced information data rate. The result shown in this paper will be helpful to explore further the multi-antenna and correlation over block coded α - μ fading channel.

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BIOGRAPHIES



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